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PBK-003-001105

Seat No. ________

B. Sc. (Sem. I) Examination

November / December - 2018

M - 101 : Geometry & Calculus

Faculty Code : 003
Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions:

(1) All questions are compulsory.
(2) Right hand side digit indicates the mark.

1 All questions are compulsory 20

(1) The Cartesian coordinate of polar point $(1, \frac{\pi}{2})$ is ________.

(2) Find the polar form of the equation $x^2 + y^2 = 4$.

(3) Write the equation of a sphere having center (a, b, c) and radius r.

(4) Find the center and radius of the sphere
$x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$.

(5) If $y = \frac{1}{x}$, then find $\frac{d^n y}{d x^n}$.

(6) $\frac{d^8 x^7}{d x^8} =$ ________.

(7) The set $\mathbb{N}$ is lower bounded. [True/False]

(8) The function $f(x) = 6(x-2)^2,\ x < 2$ is ________. (Increasing/Decreasing)

(9) Find $\lim\limits_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$.

(10) Write the indeterminate form of $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

(11) Find the integrating factor of the differential equation $\frac{dy}{dx} = x + y$.

(12) Write Clairaut's differential equation.

(13) If m_1, m_2 are distinct real roots and m_3, m_4 are distinct complex roots of auxiliary equation of homogeneous differential equation, then write the solution.

(14) $\frac{1}{D^3}3x^3 =$ ________.

(15) $(1+D)^{-1} =$ ________.

(16) Find $\int\limits_0^{\frac{\pi}{2}} \sin^{10} x dx.$

(17) Evaluate $\int\limits_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x dx.$

(18) $\frac{1}{f(D^2)} \cos ax =$ ________, provided $f(-a^2) \neq 0$.

(19) Define first order first degree homogeneous differential equation.

(20) State the alternative form of Lagrange's mean value theorem.

2 (A) Attempt any Three out of Six: 6

(1) Convert the Cartesian equation $x^3 = y^2(2a - x)$ into polar equation.

(2) Find the sphere for which $A(2, -3, 4)$ and $B(-2, 3, -4)$ are the extremities of diameter.

(3) Derive the n^{th} derivative of $\sin(ax + b)$.

(4) Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always $\frac{1}{2}$ whar ever values of p, q, r, a, h may be.

(5) Show that Maclaurin's series expansion of e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ for all $x \in \mathbb{R}$.

(6) If $f(x) = \frac{x \cos x - \log(x+1)}{x^2}$, then $\lim\limits_{x \to 0} f(x) = \frac{1}{2}$.

(B) Attempt any Three out of Six: 9

(1) Find the equation of the circle in polar coordinate system whose tangent is the initial line.

(2) Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \left(\cot^2 x - \frac{1}{x^2} \right)$.

(3) Find Approximate value of $\log_{10} 73.55$, correct up to six decimal places, where $\log_{10} 73 = 1.863323$, $\log_{10} e = 0.43429$.

(4) Derive the condition for the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ to be a sphere.

(5) Verify Lagrange's mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1, 4]$.

(6) If $y = e^{ax} \sin(bx + c)$, then find $\frac{d^n y}{d x^n}$.

(C) Attempt any Two out of Five: 10

(1) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity and find the equation sphere which has this circle for one of the great circles.

(2) State and prove Libnitz's theorem.

(3) State and prove Roll's mean value theorem.

(4) Show the $e^{m \sin^{-1} x} = 1 + mx + \frac{m^2}{2}x^2 + \frac{m(m^2+1)}{3!}x^3 + \frac{m^2(m^2+2^2)}{4!}x^4 + \cdots$.

(5) Find the relation between spherical coordinate & Cartesian coordinate. Also get the relation between cylindrical coordinate & cartesian coordinate.

3 (A) Attempt any Three out of Six: 6

(1) Solve $\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$.

(2) Solve $\frac{dy}{dx} + x \sin(2y) = x^3 \cos^2 y$.

(3) Solve $y'' - 4y' + 13y = 0$.

(4) Solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = e^{5x}$.

(5) Find the value of $\int_0^\pi \sin^6 x \cdot \cos^4 x dx$.

(6) Find the general solution of $(y - px)(p - 1) = 0$.

(B) Attempt any Three out of Six: 9

(1) Solve $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$.

(2) Solve $y - 2px = \tan^{-1}(xp^2)$.

(3) Solve $x^2(y - px) = yp^2$.

(4) Find particular integral of $(D^2 + 9)y = e^{2x} + 2x$.

(5) Find $\int \cos^6 x dx$.

(6) Prove that $\frac{1}{D-a}X = e^{ax}\int Xe^{-ax}dx$.

(C) Attempt any Two out of Five: 10

(1) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin(2x)$.

(2) Find the solution of first order linear differential equation.

(3) Find the reduction formula for $\int \sin^m x \cdot \cos^n x dx$, where $m, n \in \mathbb{N}$.

(4) Show that $\frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)V}$, where V is a function of x.

(5) State and prove the necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact.